13.3 SCHNORR DIGITAL SIGNATURE SCHEME

As with the Elgamal digital signature scheme, the Schnorr signature scheme is based on discrete logarithms [SCHN89, SCHN91]. The Schnorr scheme minimizes the message-dependent amount of computation required to generate a signature. The main work for signature generation does not depend on the message and can be done during the idle time of the processor. The message-dependent part of the signature generation requires multiplying a $2n$-bit integer with an n-bit integer.

The scheme is based on using a prime modulus p, with $p - 1$ having a prime factor q of appropriate size; that is, $p - 1 \equiv 0 \pmod{q}$. Typically, we use $p \approx 2^{1024}$ and $q \approx 2^{160}$. Thus, p is a 1024-bit number, and q is a 160-bit number, which is also the length of the SHA-1 hash value.

The first part of this scheme is the generation of a private/public key pair, which consists of the following steps.

1. Choose primes p and q, such that q is a prime factor of $p - 1$.
2. Choose an integer a, such that $a^q = 1 \bmod p$. The values a, p, and q comprise a global public key that can be common to a group of users.
3. Choose a random integer s with $0 < s < q$. This is the user's private key.
4. Calculate $v = a^{-s} \bmod p$. This is the user's public key.

A user with private key s and public key v generates a signature as follows.

1. Choose a random integer r with $0 < r < q$ and compute $x = a^r \bmod p$. This computation is a preprocessing stage independent of the message M to be signed.
2. Concatenate the message with x and hash the result to compute the value e:

$$e = \mathrm{H}(M \| x)$$

3. Compute $y = (r + se) \bmod q$. The signature consists of the pair (e, y).

Any other user can verify the signature as follows.

1. Compute $x' = a^y v^e \bmod p$.
2. Verify that $e = \mathrm{H}(M \| x')$.

To see that the verification works, observe that

$$x' \equiv a^y v^e \equiv a^y a^{-se} \equiv a^{y-se} \equiv a^r \equiv x \pmod{p}$$

Hence, $\mathrm{H}(M \| x') = \mathrm{H}(M \| x)$.